Competitive growth model involving random deposition and random deposition with surface relaxation

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A deposition model that considers a mixture of random deposition with surface relaxation and a pure random deposition is proposed and studied. As the system evolves, random deposition with surface relaxation (pure random deposition) take place with probability p and $(1-p)$, respectively. The discrete (microscopic) approach to the model is studied by means of extensive numerical simulations, while continuous equations are used in order to investigate the mesoscopic properties of the model. A dynamic scaling ansatz for the interface width $W(L,t,p)$ as a function of the lattice side L, the time t and p is formulated and tested. Three exponents, which can be linked to the standard growth exponent of random deposition with surface relaxation by means of a scaling relation, are identified. In the continuous limit, the model can be well described by means of a phenomenological stochastic growth equation with a p-dependent effective surface tension.

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I. INTRODUCTION

Over the last two decades there has been a considerable progress in the study of the morphology, structure, and other physical and chemical properties of growing interfaces [1–4]). This interest arises due to the technological applications of the subject and because evolving interfaces can be found in a great variety of physical, chemical, and biological systems and processes of scientific relevance. The growth of films either by vapor deposition, chemical deposition or molecular-beam epitaxy [1,5,6], bacterial growth [7], electrodeposition/dissolution experiments [8], propagation of reaction fronts in catalyzed reactions [9], propagation of forest fires [10], and diffusion fronts [11] are relevant examples in this area.

Models of growing interfaces may be defined and studied either on a discrete lattice or by means of continuous equations [6]. Discrete models are defined by a set of rules that provide a detailed microscopic description of the evolution of the surface. Within this context, the Family-Vicsek phenomenological scaling approach [12] has proved to be very successful for the description of the dynamic evolution of growing interfaces [1–4].

On the other hand, great progress has been achieved in the understanding of interface behavior due to the introduction of continuous equations that are focused on macroscopic aspects of surface roughness that are expected to be universal [13,14].

The study of growth models involving one kind of particle has received much attention [1–4]. However, less attention has been drawn to the study of the dynamics of competitive processes, in spite of the fact that these processes are significant to the growth of real materials in at least two different ways: (a) when the growing process involves two or more kinds of particles, and (b) when considering deposition of a single kind of particles that undergo either a deposition/evaporation process or are subjected to different growing mechanisms. One example of case (a) arises out of the deposition of alloys or systems with impurities, see, e.g., [15–20] and references therein. In this case, there may be different interactions among different kinds of particles and the growing mechanisms change [15–20]. For the example of deposition of one kind of particle (case b), Pellegrini and Jullien [21,22] have studied a ballistic model of surface growth that considers ''sticky'' and ''sliding'' particles. In a related context, very recently Shapir *et al.* [8] have reported experimental results for the surface roughness during cyclical electrodeposition/dissolution of silver.

The aim of the present work is to study a competitive dynamic process involving two different growing mechanisms, namely, random deposition (RD) and random deposition with surface relaxation (RDSR). In order to describe the scaling behavior of this model, a dynamic phenomenological scaling approach that requires two independent exponents, has been developed and tested by means of numerical simulations. Also, a phenomenological growth equation consistent with the scaling behavior observed numerically is proposed.

II. DESCRIPTION OF THE MODEL AND THE SIMULATION METHOD: DYNAMIC SCALING

A discrete growth model is studied, namely the RDSR/RD model, where particles of a single kind are aggregated either according to the rules of RDSR with probability p or according to the rules of RD with probability $(1-p)$. Numerical Monte Carlo simulations were performed in $(1+1)$ dimension using lattices of side L and taking periodic boundary conditions in the direction perpendicular to the growing surface.

In the RD growth model a column is randomly chosen along the width of the sample of length L and a particle is launched vertically until it reaches the top of the selected

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column, whereupon it is deposited. The RDSR is a variant of the RD process: a particle is released from a random position above the surface and falls vertically until reaches the top of the selected column, as in the RD model. However, the deposited particle is allowed to relax to a nearest neighbor column if the height of the neighboring column is lower than the one corresponding to the selected column.

Simulations are performed using $(1+1)$-dimensional lattices of size $64 \leqslant L \leqslant 8192$ and results are averaged over $10^2 - 10^3$ different runs, depending on L. A Monte Carlo time step (mcs) involves the deposition of L particles, so during a mcs each column grows one lattice unit (LU) in average.

The interface of the aggregate is defined as the set of particles that are placed at the highest position of each column. So, the mean height of the interface $\langle h(t) \rangle$ at time t is given by $\langle h(t) \rangle \equiv 1/L \Sigma_{i=1}^{L} h(i,t)$, where $h(i,t)$ is the height of the ith column at time t. Both, $\langle h(t) \rangle$ and $h(i,t)$ are measured in LU. In addition, the interface width $W(L,t)$, which characterizes the roughness of the interface, is defined as

$$W(L,t) \equiv \sqrt{\frac{1}{L} \sum_{i=1}^{L} [h(i,t) - \langle h(t) \rangle]^2}. \tag{1}$$

Considering a finite system and starting from a flat substrate, the width of the interface first increases algebraically according to $W(t) \propto t^{\beta}$, where $\beta > 0$ is the growth exponent of the system. As the system evolves correlations develop and eventually the correlation length reaches the size of the system. At this long-time regime the interface width saturates at some constant value $W_{sat} \propto L^{\alpha}$, where α is the roughness exponent. This type of behavior is known as the the Family-Vicsek scaling approach [12] that has proved to be very successful for the description of the dynamic evolution of a growing interface, namely,

$$W(L,t) \propto L^{\alpha} f\left(\frac{t}{L^{Z}}\right), \tag{2}$$

where Z is the dynamic exponent. In addition, $f(u)$ is a suitable scaling function that behaves as follows: (i) $f(u) =$ constant for $u \gg 1$ or, in other words, the interface width saturates for long enough times and (ii) $f(u) \propto u^{\beta}$ for $u \ll 1$. The former condition implies that $W(t) \propto t^{\beta}$ holds during the short time regime. A scaling relationship can easily be derived so that $Z = \alpha/\beta$ and only two independent exponents remain.

For RD, $W(t)$ does not saturate due the lack of lateral correlations, so $W(t) \propto t^{\beta_{RD}}$, is independent of L with $\beta_{RD} = \frac{1}{2}$. In contrast, the RDSR process causes the development of lateral correlations and, therefore, one has $\beta_{RDSR} = \frac{1}{4}$ and $\alpha_{RDSR} = \frac{1}{2}$.

III. RESULTS AND DISCUSSION

Figure 1(a) shows log-log plots of W versus t obtained for the RDSR/RD model taking $L = 256$ and using different values of p. For $p = 0$ the observed linear growth of W is characteristic of the RD process, given by $W(t) \propto t^{\beta_{RD}}$, with $\beta_{RD} = 1/2$. Considering surface relaxation $(p > 0)$, the saturation of the interface width occurs. However, the saturation value W_s sensitively depends on p: saturation takes place at longer times for smaller values of p, while the width of the interface is smaller for larger p values. Therefore, the RD process causes a twofold effect; on the one hand a delay in the propagation of correlations is observed, and on the other hand the surface roughness increases.

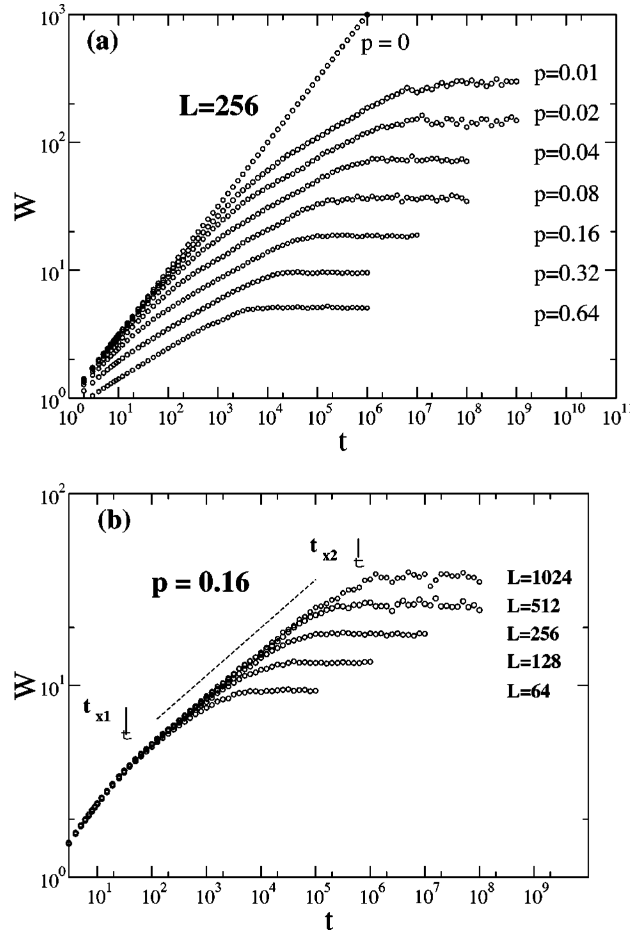

FIG. 1. Log-log plots of W versus t for the RDSR/RD model: (a) $L = 256$ and different values of p, and (b) $p = 0.16$ and lattices of different size. In (b) the arrows show the location of t_{x1} and t_{x2} for the data corresponding to $L = 1024$. Also in (b) the dashed line has slope $\beta_{RDSR} = 1/4$, and have been drawn for the sake of comparison. Distances are measured in LU and t in mcs. More details in the text.

Figure 1(b) shows plots of W versus t for lattices of different size but keeping $p = 0.16$ constant. Here, like in Fig. 1(a), three different regimes and the corresponding crossovers can easily be observed. For short times, say $t < t_{x1}$, the random growth of the interface is observed (the RD process dominates). At this stage, correlations have not developed yet and $W(t) \propto t^{\beta_{RD}} (t < t_{x1})$ holds. During an intermediate time regimen, say $t_{x1} < t < t_{x2}$, correlations develop since the RDSR process now dominates leading to $W(t) \propto t^{\beta_{RDSR}}$. At a later stage for $t > t_{x2}$, correlations can no longer grow due to the geometrical constraint of the lattice size and saturation is observed.

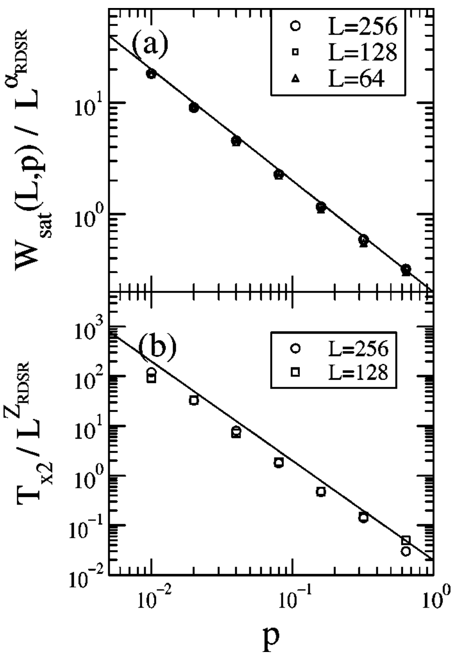

FIG. 2. (a) Log-log plots of $W_{sat}(L,p)/L^{\alpha_{RDSR}}$ versus p obtained for lattices of different size, as indicated in the figure, and assuming $\alpha_{RDSR}=1/2$. The full line has slope $\delta=0.97$ and corresponds to the best fit of the data. (b) Log-log plots of $t_{x2}/L^{Z_{RDSR}}$ versus p obtained for lattices of different size, as indicated in the figure, and assuming $Z_{RDSR}=2$. The full line has slope $y=1.97$ and corresponds to the best fit of the data. Distances are measured in LU and time in mcs. More details in the text.

Based on the data shown in Fig. 1, a phenomenological dynamic scaling approach can be proposed. Accordingly, the following ansatz for the saturation value of the interface width $[W_s(L,p)]$ and the characteristic crossover time t_{x2} is proposed

$$W_s(L,p) \propto L^{\alpha_{RDSR}} p^{-\delta} \quad (p>0), \tag{3}$$

and

$$t_{x2}(L,p) \propto L^{Z_{RDSR}} p^{-y} \quad (p>0). \tag{4}$$

In order to check the assumptions made in Eqs. (3) and (4), log-log plots of both $W_s(L,p)/L^{\alpha_{RDSR}}$ versus p [Fig. 2(a)] and $t_{x2}/L^{Z_{RDSR}}$ versus p [Fig. 2(b)], have been performed. Using the exact value $\alpha_{RDSR}=\frac{1}{2}$, straight lines are observed, in agreement with Eq. (3), and the best fit gives $\delta \cong 0.97 \pm 0.04$ for the slope in Fig. 2(a). Also, the assumption of Eq. (4) is validated by the results shown in Fig. 2(b) and the best fit of the data yields $y \cong 1.97 \pm 0.05$.

Results of extensive simulation performed using larger lattices ($L=8192$) and different values of p are shown in Fig. 3(a). It is worth mentioning that the log-log plot of W versus t shows that the (parallel) straight lines corresponding to the intermediate time regime are equally spaced and have slopes $\beta_{RDSR}=\frac{1}{4}$ (for $t>t_{x1}$). The values of p corresponding to successive curves in Fig. 3(a) differ in a factor of 2. Then, the response of the system clearly suggests a power-law behavior of the form,

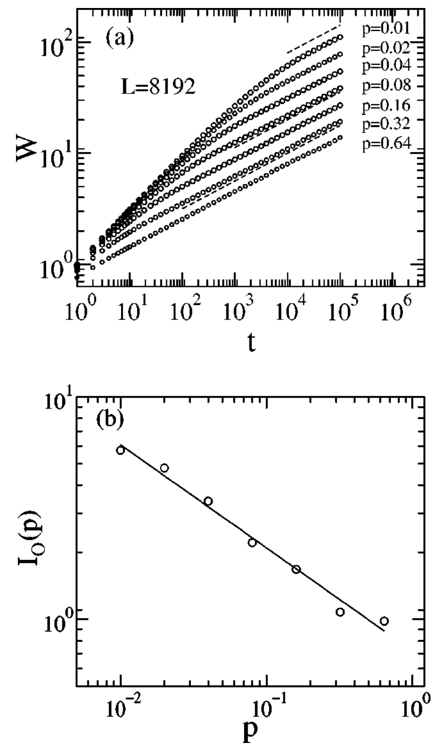

FIG. 3. (a) Log-log plot of W versus t for the RDSR/RD model obtained for $L=8192$ and different values of p as indicated in the figure. (b) Log-log plot of the ordinate intersection (I_O) obtained from Fig. 3(a) versus p. The straight line with slope $\gamma=0.51$ corresponds to the best fit of the data. Distances are measured in LU and time in mcs.

$$W(t,p) \propto t^{\beta_{RDSR}} p^{-\gamma}, \quad (t_{x1}<t<t_{x2}), \tag{5}$$

where γ is a characteristic exponent. In order to calculate this exponent, first, it is convenient to evaluate the ordinate intersection $I_O(p)$ for different values of p [see Fig. 3(a)]. Subsequently, a plot of $I_O(p)$ versus p [as shown in Fig. 3(b)] yields the value $\gamma=0.51 \pm 0.05$.

Based on all numerical results supporting Eqs. (3), (4), and (5), the following phenomenological dynamic scaling ansatz for the RDSR/RD model can be formulated

$$W(t,L,p) \propto L^{\alpha_{RDSR}} p^{-\delta} F\left(\frac{t}{L^{Z_{RDSR}} p^{-y}}\right),$$
$$p>0, \quad t>t_{x1}, \quad L\rightarrow\infty, \tag{6}$$

where $F(u)$ is a suitable scaling function which satisfies: (i) $F(u)=\text{const}$ for $u \gg 1$, so Eq. (3) for the saturation of the interface width is recovered, and (ii) $F(u)=u^{\beta_{RDSR}}$ for $u \ll 1$. It should be noticed that using the proposed ansatz, Eq. (5) can be recovered if the following scaling relationship among exponents holds

$$y\beta_{RDSR}-\delta+\gamma=0, \tag{7}$$

where the identity $Z_{RDSR}=\alpha_{RDSR}/\beta_{RDSR}$ has been used. The relationship given by Eq. (7) can be checked taking the exact value $\beta_{RDSR}=\frac{1}{4}$ and our estimations of y, δ and γ. Using

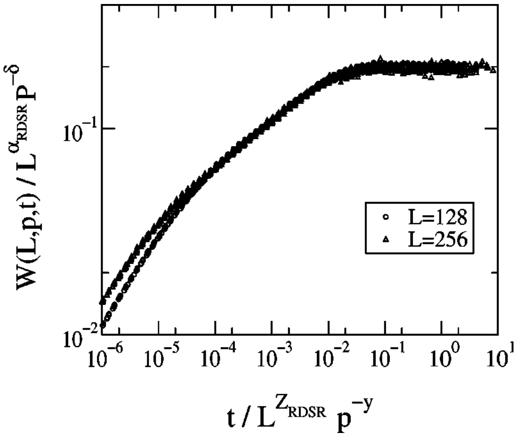

FIG. 4. Log-log plot of $W(L,t,p)/L^{\alpha_{RDSR}}p^{-\delta}$ versus $t/L^{Z_{RDSR}}p^{-y}$ obtained for different values of $p(0.01\leqslant p\leqslant 0.64)$ and lattices of size $L=256$ and $L=128$, as indicated in the figure. Distances are measured in LU and time in mcs.

these figures $y\beta_{RDR}-\delta+\gamma=0.03\pm 0.06$ is obtained, supporting the validity of Eq. (7). So, the following rational values of the exponents can be conjectured:

$$y\equiv 2 \quad (1.97\pm 0.05); \qquad \delta\equiv 1 \quad (0.97\pm 0.04);$$

$$\gamma\equiv\frac{1}{2} \quad (0.51\pm 0.05), \tag{8}$$

where the values in brackets are our numerical estimation. Of course, Eq. (7) implies that only two of the exponents are independent.

The conjectured phenomenological scaling ansatz [Eq. (6)], can be easily tested by plotting $W(L,t,p)/L^{\alpha_{RDSR}}p^{-\delta}$ versus $t/L^{Z_{RDSR}}p^{-y}$ in a log-log scale, as shown in Fig. 4. As expected, deviations from data collapsing are observed outside the range of validity of our ansatz, i.e., for $t<t_{x1}$. Otherwise, data collapsing is quite satisfactory pointing out that the proposed ansatz holds, at least as a first approach.

IV. PHENOMENOLOGICAL STOCHASTIC GROWTH EQUATION

In contrast to the microscopic details of the growing mechanisms of the interface, continuous equations focus on the macroscopic aspects of the roughness. Essentially, the aim is to follow the evolution of the coarse-grained height function $h(x,t)$ using a well-established phenomenological approach that take into account all the relevant processes that survive at a coarse-grained level. This procedure normally leads to stochastic nonlinear partial differential equations that may be written as follows [1,6,13,14]

$$\frac{\partial h(x,t)}{\partial t}=G_i\{h(x,t)\}+F+\eta(x,t), \tag{9}$$

where the index i symbolically denotes different processes, $G_i\{h(x,t)\}$ is a local functional that contains the various surface relaxation phenomena and only depends on the spatial derivatives of $h(x,t)$ since the growth process is determined

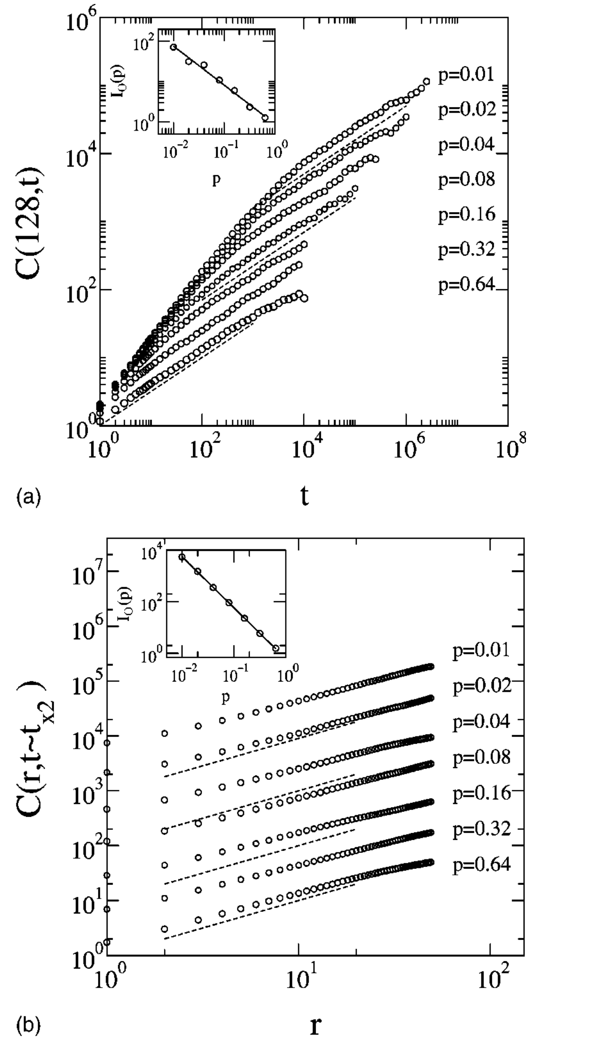

FIG. 5. (a) Log-log plots of $C(128,t)$ versus t obtained for different values of p, as indicated in the figure. The dashed-lines have slope 1/2 according to Eq. (13). The inset shows a log-log plot of the ordinate intersection (I_O) obtained from Fig. 5(a) versus p. The straight line with slope -0.97 corresponds to the best fit of the data. (b) Log-log plots of $C(r,t)$ versus r at $t\approx t_{x2}$ obtained for different values of p, as indicated in the figure. The dashed-lines have slope 1 according to Eq. (14). The inset shows a log-log plot of the ordinate intersection (I_O) obtained from Fig. 5(b) versus p. The straight line with slope -1.96 corresponds to the best fit of the data. Distances are measured in LU and time in mcs. More details in the text.

by the local properties of the surface. Also, F denotes the mean deposition rate and $\eta(x,t)$ is the deposition noise that determines the fluctuations of the incoming flux around its mean value F. It is usually assumed that the noise is spatially and temporally uncorrelated, so fluctuations are given by a Gaussian white noise.

In order to formulate a continuous description of the RDSR/RD competitive process we propose the following generalized Edwards-Wilkinson stochastic growth equation

$$\frac{\partial h(x,t)}{\partial t}=F+\nu(p)\nabla^2 h(x,t)+\eta(x,t), \tag{10}$$

where now the surface tension is assumed to depend on p. In spite of the fact that such dependence is not known explicitly, it is expected that $\nu(p)$ should be a suitable crossover function such that $\nu(p=0)=0$, so the RD model is obtained while $\nu(p=1)=\nu_o$, so the RDSR model is recovered. Equation (10) can also be solved exactly in Fourier space using a standard procedure (see, e.g., [1,23]) and after some algebra [23], the following relationships can be established

$$W^2(L,t)\propto\frac{D}{\nu(p)}L,\quad t\to\infty \text{ and large } L, \tag{11}$$

and

$$W^2(L,t)\propto t^{1/2}\nu(p)^{-1/2}L,\quad L\to\infty \text{ and large } t. \tag{12}$$

Comparing Eqs. (3) and (5) with Eqs. (11) and (12), respectively, it follows that they are consistent provided that $\nu(p)=\nu_o p^2$.

The height-height correlation function can be used to show another argument for the validity of Eq. (10). This function is given by $C(r,t)\equiv\langle(h(r,t)-h(r',t'))^2\rangle$, and describes the self-affine fluctuations of the surface height.

Solving Eq. (10) exactly in the Fourier space, we found that for $L\to\infty$ the short- and long-time behavior of the height-height correlation function can be written as

$$C(r,t)\propto\frac{D}{p}t^{1/2},\quad t\to 0, \tag{13}$$

and

$$C(r,t)\propto\frac{D}{p^2}r,\quad t\to\infty, \tag{14}$$

respectively.

In order to check these findings, the height-height correlation function has been evaluated numerically and the obtained results are shown in Fig. (5). Figure 5(a) shows that log-log plots of $C(r=128,t)$ versus t are consistent with the time dependence in Eq. (13), i.e., an exponent 1/2, as indicated by the straight-dashed lines, is obtained for a wide range of p values. Also, the ordinate intersections $[I_O(p)]$ of these straight lines behave as $I_O\propto p^{-1}$ [see the inset of Fig. 5(a)] in agreement with Eq. (13). On the other hand, Fig. 5(b) shows that log-log plots of $C(r,t_{x2})$ versus r also exhibit the power-law dependence predicted by Eq. (14). Furthermore, in this case $I_O(p)$ also behaves according to Eq. (14), as shown in the inset of Fig. (5). Summing up, we conclude that the numerical data corresponding to the height-height correlation function are in excellent agreement with the predictions of Eq. (13) and (14), supporting the validity of the proposed phenomenological continuous equation for the RDSR/RD process [Eq. (10)].

V. CONCLUSIONS

A competitive growth model, called the RDSR/RD model, that involves a mixing of random deposition (with probability $1-p$) and random deposition with surface relaxation (with probability p) is introduced and studied. The Family and Vicsek's dynamic scaling ansatz [12] is generalized and the growing interface resulting from the competitive RDSR/RD process are rationalized by means of a phenomenological dynamical scaling approach that considers three exponents, where only two of them are independent. The proposed ansatz allows us to establish a scaling relation among the exponents and the well-known growth exponent of the Edwards-Wilkinson's model.

A phenomenological continuous equation for the RDSR/RD model is proposed and solved exactly. Comparing the analytical solutions with numerical data it is concluded that the effective surface tension depends quadratically on p.

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